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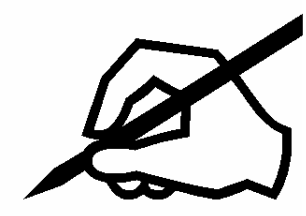

LESSON SUMMARY

CXC CSEC MATHEMATICS

UNIT Five:
Factorization

Lesson
8

Expressing Algebraic Expressions as Products

Textbook: Mathematics, A Complete Course by Raymond Toolsie, Volume 1 and 2

(Some helpful exercises and page numbers are given throughout the lesson, e.g. Ex 6g page 243)

INTRODUCTION

In this lesson we will look at factorizing algebraic expressions. This is an important technique in Mathematics. When you factorize algebraic expressions you express them as products of their factors. This is a very useful tool in solving equations and simplifying expressions.

OBJECTIVES

At the end of this lesson you will be able to:

a) Factorize algebraic expressions including; a difference of two squares, a perfect square and quadratics.

5.6 Factorization

Factorizing Using the Distributive Law

This is really like the reverse of the distributive law. Simply take outside the brackets factors that are common to all the terms in the expression.

Example:

Factorize each of the following algebraic expressions (Ex 6g page 243):

1. $9x + 9y$.

Solution:

9 is a factor of the two terms therefore take 9 outside the brackets.

$9(x + y)$.

An easy way to determine what goes into the brackets is to divide each term by the factor.

i.e. $\frac{9x}{9} = x$ and $\frac{9y}{9} = y$.

When the brackets are removed using the distributive law the original expression is obtained.

Note that the sign between the terms changes when a negative value is factored out and fractions can also be factored out.

2. $\frac{rx}{3} - \frac{ry}{6}$

Solution:

$r/3(x + y/2)$

Factorization Using the Highest Common Factor

To factorize using the H.C.F simply factor out the H.C.F of each term. If the expression consists of a variable(s) the H.C.F is the lowest power of the variable that is present in the expression.

Example:

Factorize the following (Ex 6i page 246):

$7p^3r^2 - 14pr^3$

Solution:

7 is the H.C.F of 7 and 14 and r^2 and p are the lowest powers of r and p present respectively. Therefore the H.C.F is $7pr^2$.

Hence the factorized form is

$7pr^2(p^2 - 2r)$.

Factorization by Grouping

This method of factorization is usually used when the expression has four terms. Brackets are used to group the terms in pairs. Each bracket is then factorized. Remember the sign between two terms changes when a negative sign is place outside the bracket containing them.

Example:

Factorize the following (Ex 6j page 247):

$7pl + 3ql - 7pm - 3qm$

Solution:

Use brackets to group the terms.

$(7pl + 3ql) - (7pm + 3qm)$

Factorize each bracket.

$l(7p + 3q) - m(7p + 3q)$

$(7p + 3q)$ is common to both terms now so factor that out. Therefore we are left with $(l - m)$ in a bracket by itself.

Answer:

$(7p + 3q)(l - m)$.

ACTIVITY 1

Factorize each of the following algebraic expressions:

1. $-5lx + 15ly + 25lz$

2. $\frac{p^3m^2}{7} + \frac{p^2m^3}{21} - \frac{p^4m}{14}$

3. $(3x + y)(2x - 1) - (3x + y)$

Factorization of a difference of two squares

Factorizing a difference of two squares is simple:

$a^2 - b^2 \equiv (a + b)(a - b)$.

The most difficult task is to determine what is a and what is b.

Example:

Factorize the following algebraic expression:

$4x^2 - 1$

Solution:

This is a difference of two squares since $(2[x])^2 - 1^2$. Here $a = 2x$ and $b = 1$.

Answer $= (2x + 1)(2x - 1)$.

Factorization of a perfect square

A perfect square has the form $a^2 + 2ab + b^2$ and it is factorizes as

$(a + b)(a + b) \equiv (a + b)^2$.

Example:

Factorize the following: $x^2 + 8x + 16.$ (Ex 13h page 773)

Rewrite the expression as $(x)^2 + 2(x)(4) + (4)^2.$

Clearly $a = x$ and $b = 4$. Hence the factors are $(x + 4)(x + 4) \equiv (x + 4)^2$

ACTIVITY 2

Factorize each of the following:

1. $25 - 49y^2$

2. $36x^2 + 84x + 49$

Factorization of a Quadratic

The form of a quadratic is $ax^2 + bx + c$. To factorize we have to find factors of $a \times c$ that add up to b . These factors are used to split the middle term so that factorization by grouping can be done. As always be careful with the signs.

Example:

Factorize the following quadratic expression:
$3x^2 + 5x - 2.$

Solution:

$a = 3, \qquad b = 5 \; and \; c = -2$

$a \times c = 3 \times -2$

$-6.$

Factors of -6 that add up to 5 is 6 and -1 . These factors will be used to split the middle term.

Therefore we have $3x^2 + 6x - x - 2$.

Now factorize by grouping.

$(3x^2 + 6x) - (x + 2)$

$3x(x + 2) - 1(x + 2)$

By factoring out $x + 2$ we have:

$(x + 2)(3x - 1)$.

Factorize the following quadratic expressions: (Ex 13j page 778)

1. $28 + 3x - x^2$
2. $6x^2 - 11x - 35$

CXC Questions

Factorize completely:

(i) $3xy - x$

(ii) $4a^2 - 9$

(iii) $(x - y)^2 - x + y$

(iv) $6x^2 - 13x - 5$

CONCLUSION

We have looked at the factorization of a number of algebraic expressions. These include quadratics. In the lesson that follows we will look at completing the square and the application of factorization. There you will find the skills acquired in this lesson very useful.